Fourth Generation Leptons and Muon $g-2$

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We consider the contributions to $g_\mu - 2$ from fourth generation heavy neutral and charged leptons, N and E, at the one-loop level. Diagrammatically, there are two types of contributions: boson-boson-N, and E-E-boson in the loop diagram. In general, the effect from N is suppressed by off-diagonal lepton mixing matrix elements. For E, we consider flavor changing neutral couplings arising from various New Physics models, which are stringently constrained by $\mu \to e\gamma$. We assess how the existence of a fourth generation would affect these New Physics models.

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I. INTRODUCTION

The fourth generation has been viewed as out of favor [1] since a long time, because of electroweak precision tests (EWPrT), and neutrino counting on the Z peak. However, the severeness of the S parameter constraint from EWPrT has been questioned recently [2], while we know that the neutrino sector is much richer than originally thought because of neutrino oscillations. With the advent of the LHC, we now have a machine which can discover or rule out the 4th generation by direct search, once and for all [3]. Currently, the Tevatron has set stringent limits [4] on t' via $t' \to qW$ search.

It was recently pointed out [5] that the existence of a 4th generation could have implications for the baryon asymmetry of the Universe (BAU). By shifting the Jarlskog invariant [6] for CP violation (CPV) of the 3 generation Standard Model (SM3) by one generation, i.e. from 1-2-3 to 2-3-4 quarks, one gains by more than 10^{13} in effective CPV, and may be sufficient for BAU! Recent developments in CPV studies at the B factories [7] and the Tevatron [8] suggest the 4th generation could be behind some hints for New Physics in $b \to s$ transitions. From a different perspective, whether from effective 4-fermion interactions [9], or from holographic extra dimension considerations [10] (the two are complementary), there are also recent interest in very heavy 4th generation quarks, where their heaviness could be responsible for inducing electroweak symmetry breaking itself.

With renewed interest in the existence of a sequential 4th generation, and with experimental discovery or refutation expected at the LHC in due time, we turn to the lepton sector. Our goal is modest: if a 4th generation exists, what are the implications for the most prominent probes with charged leptons, i.e. muon $g-2$, $\mu \to e\gamma$, and $\tau \to \ell\gamma$?

The difference between the experimental value and the SM3 prediction of muon $g-2$ has been around for some time now [11]. That is,

$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = 295(88) \times 10^{-11}, \qquad (1)$$

where $a_\mu \equiv (g_\mu - 2)/2$. The difference of over 3.4σ has aroused a lot of interest. We also have very stringent bounds on lepton flavor violating (LFV) rare decays, such as [1]

$$\mathcal{B}(\mu \to e\gamma) < 1.2 \times 10^{-11}, \qquad (2)$$

and the τ decay counterpart [12]

$$\mathcal{B}(\tau \to e\gamma) < 1.1 \times 10^{-7}, \qquad (3)$$

$$\mathcal{B}(\tau \to \mu\gamma) < 4.5 \times 10^{-8}, \qquad (4)$$

at 90% C.L. These limits could be improved further in the near future. The MEG experiment, a $\mu \to e\gamma$ search experiment aiming at a sensitivity of 10^{-13} [13], has started its physics run in 2008. The short 2008 run alone is expected to bring the limit below 10^{-12}. Though the limits on $\tau \to \ell\gamma$ from the B factories, Eqs. (3) and (4), will soon be limited by B factory statistics, a Super B Factory upgrade could push down to the 10^{-8} region, which become background limited (for outlook, see Ref. [14]).

Can the effect of 4th generation leptons show up in the probes of Eqs. (1)–(4)? How would these processes constrain New Physics models in the presence of a 4th generation? In this paper we start our discussion from a diagrammatic point of view, and in so doing, correct some errors in the literature.

The 4th generation neutral lepton N can enter the loop with charged vector boson $W^\pm$ or scalar boson $H^\pm$, which we plot for $\mu\mu\gamma$ coupling in Fig. 1(a) for illustration. The diagrams for $\mu \to e\gamma$ and $\tau \to \ell\gamma$ are quite similar. The W^+W^-N loop is controlled by the lepton mixing matrix elements, while the H^+H^-N loop may become important because of m_N. The charged lepton E can enter the loop with neutral scalar and pseudo scalar bosons h^0 and A^0, or a neutral vector boson Z', as illustrated in Fig. 1(b). However, these neutral bosons would

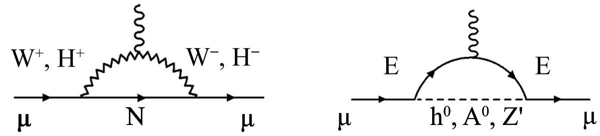

Fig. 1: (a) Boson-boson-N and (b) E-E-boson loop diagrams.

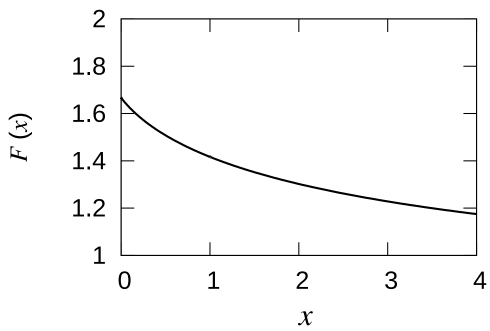

Fig. 2: Loop function $F(x)$ of Eq. (5), vs $x = m_N^2/M_W^2$.

need to have flavor changing neutral couplings (FCNC, absent in SM) to be relevant. But then they will face stringent constraints from $\mu \to e\gamma$ and $\tau \to \ell\gamma$. Therefore, if the 4th generation exists, Eqs. (1)–(4) will constrain New Physics models.

In the next section we first discuss the contributions involving neutral lepton N. In Sec. III we discuss the contributions involving charged lepton E. In Sec. IV we compare with the minimal supersymmetric SM (MSSM). A summary is given in Sec. V.

II. EFFECTS OF NEUTRAL LEPTON N

The 4th generation neutral lepton N enters the one-loop diagram for muon $g-2$ illustrated in Fig. 1(a). The boson can be the charged vector boson $W^\pm$, or charged scalar boson $H^\pm$, which we discuss separately.

A. W^+W^-N Loop Contribution

With ν_μ instead of N, this is the only contribution within SM. The contribution from a fourth generation lepton N has been considered before [15, 16]. We find

$$a_\mu(W^+W^-N) \;=\; \frac{G_{\rm F} m_\mu^2}{4\sqrt{2}\pi^2}\,|V_{N\mu}|^2\,F(x), \qquad (5)$$

where $x = m_N^2/M_W^2$, $V_{N\mu}$ is the lepton mixing matrix element, and

$$\begin{aligned} F(x) \;&=\; \int_0^1 du\,\frac{u(1-u)(2-u)x + 2u^2(u+1)}{(1-u)x+u} \\ &=\; \frac{3x^3\log x}{(x-1)^4} + \frac{4x^3 - 45x^2 + 33x - 10}{6(x-1)^3}. \qquad (6) \end{aligned}$$

We depict $F(x)$ versus x in Fig. 2. We see that $F(x)$, an Inami–Lim [17] loop function, is well-behaved and bounded, with $F(1) = 17/12$. However, this does not seem to be correctly rendered in Refs. [15] and [16]. In Ref. [15], though bounded, $F(x)$ was not correctly evaluated. Furthermore, the authors found strong enhancement near $x \sim 1$, which was used to put a bound on the fourth neutral lepton mass. However, from the integral in Eq. (6) and the functional form of $F(x)$, it should be clear that there is no enhancement near $x = 1$ (i.e. $m_N \sim M_W$). In Ref. [16], which is a study note for Ref. [18], the form of $F(x)$ is again incorrect but still bounded. The author claimed that $F(x)$ had a singularity at $x = 1$, and attributed this to the zero width approximation of the W boson propagator. Again, we see from Eq. (6), that there is no singularity for any x, and in any case, Γ_W should be irrelevant for such low scale processes. Our result therefore corrects some errors in the literature [19].

As we have already stated, if we replace N by ν_μ, we should recover the SM contribution. Using $F(0) = 5/3$ in Eq. (5), together with $V_{\nu_\mu\mu} \cong 1$, we get $a_\mu^{\rm SM}(W^+W^-\nu_\mu) = 5G_{\rm F}m_\mu^2/12\sqrt{2}\pi^2$. Furthermore, we find $a_\mu^{\rm SM}(\mu\mu Z) = -(G_{\rm F}m_\mu^2/6\sqrt{2}\pi^2)(1 + 2\sin^2\theta_W - 4\sin^4\theta_W)$, where $\sin^2\theta_W = 0.23120$. Combining the two together, and using

$$\frac{G_F m_\mu^2}{4\sqrt{2}\pi^2} \;\simeq\; 233\times 10^{-11}, \qquad (7)$$

we get $a_\mu^{\rm SM}$(1-loop Electroweak) $= 195\times 10^{-11}$, which is consistent with Ref. [20].

The SM exercise indicates that the 4th generation neutral lepton contribution has the right order of magnitude to contribute to Eq. (1). However, as seen from Fig. 2, the effect actually drops a bit from the massless ν_μ result of SM as the mass of N becomes heavier. Furthermore, it is multiplied by the suppression factor $|V_{N\mu}|^2$. From $m_N \gtrsim 90$ GeV [1], hence $F(x) \lesssim 1.4$, we see that $|V_{N\mu}|$ needs to be 0.7 or higher to reach within 2σ of Eq. (1). Considering the stringent constraint from Eq. (2), however, this is clearly unrealistic. We conclude that the difference of Eq. (1) cannot come from the addition of a 4th neutral lepton N.

B. H^+H^-N Loop Contribution

It is unusual to consider both a 4th neutral lepton N together with charged Higgs H^+. But since W^+W^-N contribution is insufficient for Eq. (1), we consider replacing W^+ by the charged Higgs H^+. This is the Two-Higgs-Doublet-Model (2HDM) with 4th generation leptons. It is of interest to check whether one could gain from large $\tan\beta$ enhancement.

For 2HDM-II (which occurs for MSSM), where up and down type quarks receive masses from different Higgs doublets, we find

$$\begin{aligned} &a_\mu^{\rm 2HDM-II}(H^+H^-N) \\ &= -\frac{G_F\,m_\mu^2}{4\sqrt{2}\pi^2}|V_{N\mu}|^2\,[f_{H^+}(x) \\ &\qquad +g_{H^+}(x)\cot^2\beta + x_\mu\,q_{H^+}(x)\tan^2\beta], \qquad (8) \end{aligned}$$

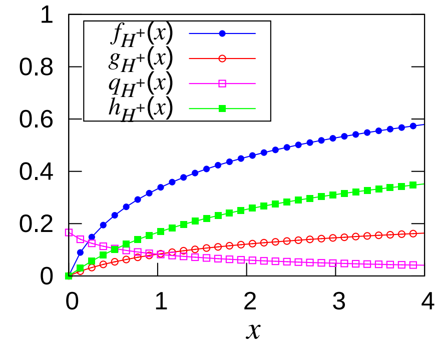

Fig. 3: Loop functions $f_{H^+}(x)$, $g_{H^+}(x)$, $q_{H^+}(x)$ of Eq. (8), and $h_{H^+}(x)$, $q_{H^+}(x)$ of Eq. (12) vs $x = m_N^2/M_{H^+}^2$.

where $x = m_N^2/M_{H^+}^2$ and $x_\mu = m_\mu^2/M_{H^+}^2$. The loop functions in Eq. (8) are

$$f_{H^+}(x) = \int_0^1 du\, \frac{2u(1-u)x}{(1-u)\,x+u} = -\frac{2x^2\log x}{(x-1)^3} + \frac{x(x+1)}{(x-1)^2}, \tag{9}$$

$$g_{H^+}(x) = \int_0^1 du\, \frac{u^2(1-u)x}{(1-u)\,x+u} = -\frac{x^3\,\log\,x}{(x-1)^4} + \frac{x(2x^2+5x-1)}{6(x-1)^3}, \tag{10}$$

$$q_{H^+}(x) = \int_0^1 du\, \frac{u^2(1-u)}{(1-u)\,x+u} = -\frac{x^2\,\log\,x}{(x-1)^4} + \frac{2\,x^2+5\,x-1}{6(x-1)^3}. \tag{11}$$

We plot $f_{H^+}(x)$, $g_{H^+}(x)$ and $q_{H^+}(x)$ in Fig. 3. The $q_{H^+}(x)$ term in Eq. (8) can actually be safely ignored, because one would need extremely large values of $\tan\beta$ to overcome the extremely small x_μ. However, we give a complete expression to check a previous result in Ref. [21] for 3 generations. If we replace N be ν_μ, one has $f_{H^+}(0) = 0$, $g_{H^+}(0) = 0$ and $q_{H^+}(0) = 1/6$, and Eq. (8) becomes $a_\mu^{\rm 2HDM-II}(H^+H^-\nu_\mu) = -(G_F m_\mu^2/4\sqrt{2})x_\mu\, \tan^2\beta/6$.

For the first term of Eq. (8), for $1 \lesssim x \lesssim 10$ we have $0.4 \lesssim f_{H^+}(x) \lesssim 0.8$, which is not particularly small. But because of the general $|V_{N\mu}|^2$ suppression, an argument similar to the W^+W^-N loop discussion suggest that this term can not give rise to Eq. (1). It is interesting that, because N has isospin $+1/2$, large $\cot\beta$ could lead to enhancement. If we take $|V_{N\mu}\cot\beta|^2$ to be order 1 in the large $\cot\beta$ limit, and if m_N is large compared to m_{H^+}, it could generate a finite contribution. But this contribution is *negative*, hence it is in the wrong direction for Δa_μ of Eq. (1). Furthermore, in the 2HDM-II (or MSSM), the $t\bar{t}H^0(h^0)$ coupling relative to its SM value, m_t/v, is given by $\cos\alpha/\sin\beta$ $(\sin\alpha/\sin\beta)$. Large $\cot\beta$ will make the coupling strength $|g_{t\bar{t}H^0}| \gg 1$ or $|g_{t\bar{t}h^0}| \gg 1$ and become nonperturbative, which is not desirable.

It has already been pointed, however, that there is a sufficient contribution in MSSM [20, 22, 23] coming from the large $\tan\beta$ region. We will give a brief assessment in Sec. IV.

For 2HDM-I, where all quarks receive mass from the same Higgs doublet, we find

$$a_\mu^{\rm 2HDM-I}(H^+H^-N) = \frac{G_F\, m_\mu^2}{4\sqrt{2}\pi^2}|V_{N\mu}|^2\cot^2\beta[h_{H^+}(x) - x_\mu\, q_{H^+}(x)], \tag{12}$$

with x and x_μ as before, and

$$h_{H^+}(x) = \int_0^1 du\, \frac{u(1-u)(2-u)x}{(1-u)x+u} = -\frac{x^2(x-2)\log\,x}{(x-1)^4} + \frac{x(4\,x^2-5\,x-5)}{6(x-1)^3}, \tag{13}$$

while $q_{H^+}(x)$ is given in Eq. (11). We plot $h_{H^+}(x)$ also in Fig. 3. Analogous to 2HDM-II, we have $a_\mu^{\rm 2HDM-I}(H^+H^-\nu_\mu) = -(G_F m_\mu^2/4\sqrt{2})x_\mu\cot^2\beta/6$, but now everything is proportional to $\cot^2\beta$.

Similar to the 2HDM-II case, if we take $|V_{N\mu}\cot\beta|^2$ to be order 1, it could generate a finite and *positive* contribution to Δa_μ. However, for 2HDM-I, the $\cot\beta$ enhanced Higgs couplings to $t\bar{t}$ are non-perturbative at large $\cot\beta$, which leads us to reject this possibility.

III. EFFECTS OF CHARGED LEPTON E

The 4th generation charged lepton E contributes to a_μ via E-E-boson loop diagrams, where the boson has to be neutral, and can be a scalar (h^0), pseudo-scalar (A^0), or vector (an extra Z'). But they need to possess flavor changing neutral couplings (FCNC). We discuss each case separately.

A. EEh^0, EEA^0 Loop Contribution

There is no μEH^0 coupling in SM. The same is true for the 2HDM-I and II, and μEH^0, μEh^0 and μEA^0 couplings are absent. This is because, by design [24], the charged leptons receive mass from just one doublet, and only one matrix needs to be diagonalized. In the so-called 2HDM-III, this restriction is softened, and there exist two matrices $\eta^{e(\nu)}$ and $\xi^{e(\nu)}$ simultaneously for each lepton type. Note that in this model, by redefining ϕ_1, ϕ_2 and η, ξ simultaneously, which still leaves the Lagrangian invariant, we may assume $\langle\phi_1^0\rangle = v/\sqrt{2}$ and $\langle\phi_2^0\rangle = 0$ without loss of generality, hence $\tan\beta$ is no longer a physical parameter. For a detailed analysis, we refer to Ref. [25].

To regulate the FCNC in face of stringent constraints, there is the ansatz suggested by Cheng and Sher [26] for the quark sector, i.e. all $q_iq_jh^0/H^0/A^0$ couplings have the same form

$$\Delta_{ij}\frac{\sqrt{m_im_j}}{v}, \tag{14}$$

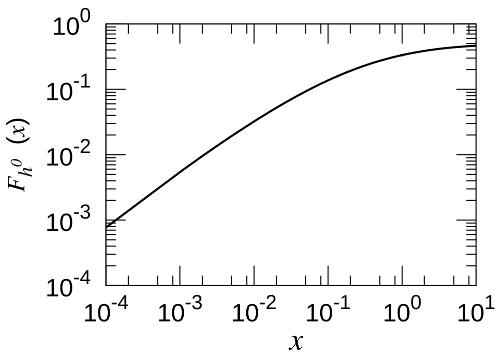

Fig. 4: Loop function $F_{h^0}(x)$ of Eq. (15), vs $x = m_E^2/M_{h^0}^2$.

where Δ_{ij} is of $\mathcal{O}(1)$. This scheme can survive the rather critical constraint of K^0–$\bar{K}^0$ mixing, because $\sqrt{m_d m_s}/v$ is extremely small. We extend it here to the charged lepton sector with 4th generation, although it may not hold because the lepton mixing pattern seems different from those of the quarks.

Note that CP-even Higgs bosons H^0, h^0 give *positive* contributions to a_μ, but CP-odd A^0 contributions are *negative*. Considering the positivity of Eq. (1), we may assume A^0 is very heavy hence can be safely neglected. For sake of illustration, we set h^0 to be the lightest neutral Higgs, and assume no mixing between H^0 and h^0. We then find

$$\Delta a_\mu^{\rm 2HDM-III} \sim 233 \times 10^{-11}\, F_{h^0}(x), \tag{15}$$

where $x = m_E^2/M_{h^0}^2$ and we have taken $\Delta_{ij} = 1$, and

$$\begin{aligned} F_{h^0}(x) &= \int_0^1 du\, \frac{u^2\, x}{u\, x + (1-u)} \\ &= \frac{x \log x}{(x-1)^3} + \frac{x(x-3)}{2(x-1)^2}. \end{aligned} \tag{16}$$

which is plotted in Fig. 4.

There are other loops such as H^+H^-N to be considered, but they are suppressed by m_μ/m_E and can be safely neglected. The suppression factor is m_μ/m_E rather than $(m_\mu/m_E)^2$ because of the Cheng-Sher coupling enhancement in Eq. (14). However, the LFV decay rates in Eqs. (2)–(4) give very stringent constraints, and need to be confronted. Here we use the formulas in Ref. [27]. Note that because a_μ and $\mathcal{B}(\mu \to e\gamma)$ come from loop diagrams of similar structure, their formulas are very closely related. After some organization, we have

$$\begin{aligned} \mathcal{B}^{\rm 2HDM-III}(\mu \to e\gamma) &= \frac{3\alpha}{2\pi}\frac{m_e}{m_\mu}|F_{h^0}(x)|^2 \\ &= 1.7 \times 10^{-5}|F_{h^0}(x)|^2\,. \end{aligned} \tag{17}$$

Let us consider first the case of τ in the loop, which is the leading contribution with 3 generations, and was discussed in Ref. [28]. Eq. (17) becomes

$$\begin{aligned} &\mathcal{B}^{\rm 2HDM-III}(\mu \to e\gamma)|_{\rm 3\ gen.} \\ &= \frac{3\alpha}{2\pi}\frac{m_e}{m_\mu}\left|\frac{m_\tau^2}{M_{h^0}^2}\left(\log\frac{m_\tau^2}{M_{h^0}^2} + \frac{3}{2}\right)\right|^2. \end{aligned} \tag{18}$$

Considering a factor of 2 uncertainty in Eq. (18), we still require $M_{h^0} > 138$ GeV in order to survive Eq. (2). We note that the MEG experiment can push the lower bound down to 530 GeV.

Consider now 4 generations. Comparing Eqs. (1) and (15), we have $F_{h^0}(x) = \mathcal{O}(1)$, and it seems that one could in principle bring about the difference with 4th generation under Cheng–Sher ansatz. However, Eqs. (2) and (17) give $F_{h^0}(x) \lesssim 10^{-3}$. From Fig. 4, we see that $m_E \ll M_{h^0}$ is necessary, which is in conflict with data. Thus, the $\mu \to e\gamma$ constraint rules out the Cheng–Sher ansatz with 4th generation lepton in the loop.

Even if we neglect the above shortfall, i.e. if we assume e decouples from Eh^0, we still face the constraint on $\mathcal{B}(\tau \to \mu\gamma)$, i.e.

$$\begin{aligned} &\mathcal{B}^{\rm 2HDM-III}(\tau \to \mu\gamma) \\ &= \mathcal{B}(\tau \to \mu\bar{\nu}_\mu\nu_\tau)\frac{3\alpha}{2\pi}\frac{m_\mu}{m_\tau}|F_{h^0}(x)|^2 \\ &= 3.6 \times 10^{-5}|F_{h^0}(x)|^2. \end{aligned} \tag{19}$$

If we hold that $m_E/M_{h^0} > 0.1$ as reasonable, then $\tau \to \mu\gamma$ is again ruled out by Eq. (4) already.

If the 4th generation is found, it seems that the Cheng-Sher ansatz can not hold for the lepton sector.

B. EEZ' Loop Contribution

For completeness, we discuss EEZ' contribution. Z' is the new gauge boson associated with an additional Abelian gauge symmetry $U'(1)$ [29]. Because the typical constraint on the Z–Z' mixing angel θ is $\theta < \mathcal{O}(10^{-3})$, we assume for simplicity that there is no mixing between Z' and Z, i.e. they are also the mass eigenstates. In terms of physical fields, the Lagrangian associated with the $U'(1)$ gauge symmetry in the charged lepton sector is written as

$$\mathcal{L}_{Z'} = g_{Z'}\bar{e}_a\gamma^\mu\left[\epsilon_{ab}^L\, L + \epsilon_{ab}^R\, R\right] e_b\, Z'_\mu, \tag{20}$$

where ϵ_{ab}^L and ϵ_{ab}^R are the 4×4 chiral coupling matrices of Z' with charged leptons $a, b = 1, ..., 4$ are flavor indices, and $L(R)$ is the left(right)-handed projection operator $(1 \mp \gamma^5)/2$. Because of the reality of the Lagrangian, $\epsilon^{L(R)}$ must be hermitian.

After some calculation, we get the dominant contribution to $g_\mu - 2$ from EEZ' loop diagram,

$$\begin{aligned} a_\mu(EEZ') &= \frac{g_{Z'}^2}{8\pi^2}\Big[x_\mu^{1/2}\,{\rm Re}\left(\epsilon_{\mu E}^L\epsilon_{E\mu}^R\right) f_Z(x) \\ &\quad - x_\mu\left(|\epsilon_{\mu E}^L|^2 + |\epsilon_{\mu E}^R|^2\right) q_Z(x)\Big], \end{aligned} \tag{21}$$

where $x = m_E^2/M_{Z'}^2$ and $x_\mu = m_\mu^2/M_{Z'}^2$. The loop functions are

$$
\begin{aligned}
f_Z(x) &= \int_0^1 du \frac{4u(1-u)x^{1/2} + u^2 x^{3/2}}{u\,x + (1-u)} \\
&= -\frac{3x^{3/2}\log x}{(x-1)^3} + \frac{x^{1/2}(x^2+x+4)}{2(x-1)^2}, \quad (22) \\
q_Z(x) &= \int_0^1 du\, \frac{u(1-u)(2-u)}{(1-u)+u\,x} \\
&= -\frac{x(2x-1)\log x}{(x-1)^4} + \frac{5x^2+5x-4}{6(x-1)^3}, \quad (23)
\end{aligned}
$$

which are plotted in Fig. 5.

As a check, we take the case of $\mu\mu Z$. Then $g_Z = e/\sin\theta_W\cos\theta_W$, $\epsilon^L_{\mu\mu} = 1/2 - \sin^2\theta_W$ and $\epsilon^R_{\mu\mu} = -\sin^2\theta_W$. From Eqs. (22) and (23) we have $f_Z(x_\mu) = 2\sqrt{x_\mu}$ and $q_Z(x_\mu) = 2/3$. Hence, we find $a_\mu(\mu\mu Z) = -(G_F\, m_\mu^2/6\sqrt{2}\pi^2)(1+2\sin^2\theta_W - 4\sin^4\theta_W)$, which is consistent with Ref. [20], as mentioned in Sec. II-A.

On the other hand, we can also get the contribution to the branching ratio of $\mu \to e\gamma$ from EEZ' loop diagram,

$$
\begin{aligned}
\mathcal{B}^{Z'}(\mu \to e\gamma) &= \frac{3\alpha g_{Z'}^4}{8\pi\, G_F^2\, m_\mu^2\, M_{Z'}^2} |f_Z(x)|^2 \\
&\quad \times \Big[|\epsilon^R_{eE}\epsilon^L_{E\mu}|^2 + |\epsilon^L_{eE}\epsilon^R_{E\mu}|^2 \Big]. \quad (24)
\end{aligned}
$$

For $\tau\tau Z'$ in the loop, we find

$$
a_\mu(\tau\tau Z') = \frac{g_{Z'}^2}{4\pi^2}\frac{m_\mu m_\tau}{M_{Z'}^2} \mathrm{Re}\left(\epsilon^L_{\mu\tau}\epsilon^R_{\tau\mu}\right), \quad (25)
$$

which is given in Ref. [30], but with a sign error. A similar formula holds for E in the loop. It can be seen that if $\epsilon^L_{\mu E}$ or $\epsilon^R_{\mu E}$ is zero (purely right-handed or purely left-handed), $a_\mu(EEZ')$ will become insignificant. This is because a spin flip is required.

For sake of illustration, we take $M_{Z'} = 1\,\mathrm{TeV}$, $m_E = 250\,\mathrm{GeV}$, $g_{Z'} = 0.105$ (predicted from a string model [31]), and denote $\epsilon^L_{\mu E} = \epsilon^R_{\mu E} \equiv \epsilon_{\mu E}$ and real, then

$$
a_\mu(EEZ') \sim 620 \times 10^{-11}\,\epsilon^2_{\mu E}. \quad (26)
$$

Furthermore, we assume $\epsilon^L_{eE} = \epsilon^R_{eE} \equiv \epsilon_{eE}$ and real, then

$$
\mathcal{B}^{Z'}(\mu \to e\gamma) \sim 2.47 \times 10^{-2}\ \epsilon^2_{eE}\,\epsilon^2_{\mu E}. \quad (27)
$$

Comparing Eq. (1) with Eq. (26), we need $\epsilon_{\mu E} = \mathcal{O}(1)$ for Z' to be the main contributor to muon $g-2$. This is reasonable for a gauge interaction, as already stated. But one would need a model for why $\epsilon_{\mu E}$ is of order one. Furthermore, comparing Eqs. (2) and (27), we find $\epsilon_{eE}/\epsilon_{\mu E} = \mathcal{O}(10^{-4})$ to satisfy $\mu \to e\gamma$ constraint. Hence, the model would not only need to account for $\epsilon_{\mu E} \sim 1$, but $\epsilon_{eE} \ll 1$ as well. If this Z' is a main contributor to muon $g-2$, it better not couple to electrons.

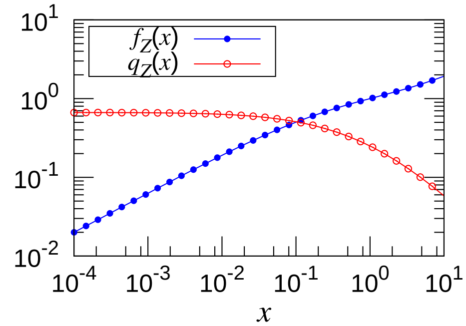

Fig. 5: Loop functions $f_Z(x)$ and $q_Z(x)$ of Eq. (21), vs $x = m_E^2/M_{Z'}^2$.

For completeness, we discuss the contribution to the branching ratio of $\tau \to \mu\gamma$ from EEZ' loop diagram. Using a similar formula to Eq. (24), we find

$$
\begin{aligned}
&\mathcal{B}^{Z'}(\tau \to \mu\gamma) \\
&= \mathcal{B}(\tau \to \mu\bar{\nu}_\mu\nu_\tau)\frac{3\alpha g_{Z'}^4}{8\pi\, G_F^2\, m_\tau^2\, M_{Z'}^2}|f_Z(x)|^2 \\
&\quad \times \Big[|\epsilon^R_{\mu E}\epsilon^L_{E\tau}|^2 + |\epsilon^L_{\mu E}\epsilon^R_{E\tau}|^2 \Big]. \quad (28)
\end{aligned}
$$

If we take the same assumption and denote $\epsilon^L_{\tau E} = \epsilon^R_{\tau E} \equiv \epsilon_{\tau E}$ and real, then

$$
\mathcal{B}^{Z'}(\tau \to \mu\gamma) \sim 1.52 \times 10^{-5}\ \epsilon^2_{\mu E}\,\epsilon^2_{\tau E}. \quad (29)
$$

Comparing Eq. (4) with Eq. (29), taking $\epsilon_{\mu E} \sim 1$ as before, we find $\epsilon_{\tau E} \sim 10^{-2}$ is needed to satisfy $\tau \to \mu\gamma$ constraint. Thus, it does not seem likely that the Z' is the dominant source for muon $g-2$.

IV. COMPARISON WITH MSSM

From previous discussions, we understand that the loop function (Inami–Lim functions) does not receive significant enhancement with heavy particle in the loop, so the coupling strengths become the crucial factors. For example, the magnitudes of electroweak contribution $a_\mu(W^+W^-\nu_\mu)$ and $a_\mu(\mu\mu Z)$ both enter the interesting range $295(88) \times 10^{-11}$ because they just involve gauge couplings, without any off-diagonal suppression. On the other hand, extreme smallness of $\mathcal{B}(\mu \to e\gamma)$ is predicted in the framework of SM, since there is no tree level FCNC, while loop effects are highly GIM suppressed by neutrino mass. Are *other* sizable electroweak contribution possible? As we mentioned in Sec. II-B, there is such a mechanism [20, 22, 23] in the MSSM.

Simply put, MSSM doubles the number of diagrams of SM. The corresponding loops to $W^+W^-\nu_\mu$ and $\mu\mu Z$ are chargino-chargino-$\tilde{\nu}_\mu$ and $\tilde{\mu}$-$\tilde{\mu}$-neutralino respectively. Assuming mass degeneracy of superparticles, $m_{\mathrm{Higgsino}} = m_{\mathrm{Wino}} = M_{\tilde{\nu}_\mu} = M_{\mathrm{SUSY}}$, and in the large $\tan\beta$ limit (to

compensate the extra heaviness of $M_{\rm SUSY}$) [22], one can get a sufficient contribution.

However, Ref. [20] used a different degeneracy condition, $m_{\rm chargino} = M_{\tilde{\nu}_\mu} = M_{\rm SUSY}$, which would inadvertently send the chargino loop into a suppression region. Because it is a little subtle, we take a closer look.

Following the formulas in Ref. [22] and under the condition $m_{\rm chargino} = M_{\tilde{\nu}_\mu} = M_{\rm SUSY}$, we have

$$\sum_i a_\mu(\chi_i^+\chi_i^-\tilde{\nu}_\mu) \;=\; \frac{1}{48\pi^2}\sum_i \Big[x_\mu(C_i^{L*}C_i^L + C_i^{R*}C_i^R) -9x_\mu^{1/2}{\rm Re}(C_i^{L*}C_i^R)\Big]\,, \quad (30)$$

with $x_\mu = m_\mu^2/M_{\rm SUSY}^2$, $C_i^L = (\sqrt{2}m_\mu/v\cos\beta)(U_{\chi^-})_{2i}$ and $C_i^R = -g_2(U_{\chi^+})_{1i}$, while U_{χ^+} and U_{χ^-} are unitary matrices and related by

$$\begin{pmatrix} -M_{G_2} & \frac{g_2 v\cos\beta}{\sqrt{2}} \\ -\frac{g_2 v\sin\beta}{\sqrt{2}} & \mu_H \end{pmatrix} = M_{\rm SUSY}\, U_{\chi^+}U^\dagger_{\chi^-}. \quad (31)$$

After some calculation, we get

$$\sum_i a_\mu(\chi_i^+\chi_i^-\tilde{\nu}_\mu) \sim 837\times 10^{-5}x_\mu. \quad (32)$$

With $x_\mu = m_\mu^2/M_{\rm SUSY}^2 \lesssim (10^{-7})$ typically, the chargino-chargino-$\tilde{\nu}_\mu$ loop contribution is subdominant.

V. SUMMARY

In this paper, we consider the existence of 4th generation leptons and discussed their impact on a_μ. In the SM, 2HDM-I and II, the 4th generation seems irrelevant to the Δa_μ puzzle because of the smallness of $|V_{N\mu}|$. However, this off-diagonal factor also protects these models from the stringent $\mu\to e\gamma$ and $\tau\to\mu\gamma$ constraints. In the 2HDM-III, applying the Cheng–Sher ansatz with 4th generation to charged leptons, one has a strong conflict with $\mathcal{B}(\mu\to e\gamma)$ and even $\mathcal{B}(\tau\to\mu\gamma)$. Hence, if 4th generation is found, the Cheng–Sher ansatz cannot hold for lepton sector. This may be reasonable since the lepton mixing pattern seems different from quarks.

Our analysis illustrates why the well known SUSY mechanism is favored. Enhancement to a_μ and suppression to $\mathcal{B}(\mu\to e\gamma)$ in the MSSM both bear similarities to the SM. It is interesting that the required large $\tan\beta$ enhancement for the SUSY effect renders the *negative* contribution from H^+H^-N negligible, hence MSSM and 4th generation can coexist.

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